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# COVARIANT FIRST AND SECOND QUANTIZATION OF THE $N = 2$, $D = 10$ BRINK-SCHWARZ SUPERPARTICLE

E. NISSIMOV*, S. PACHEVA* and S. SOLOMON

*Department of Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

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Using a new type of harmonic superspace variables, we reduce the $N=2$, $D=10$ Brink-Schwarz (BS) superparticle to a system whose constraints are (i) first class, (ii) functionally independent and (iii) Lorentz covariant. We show that these features are essential for a correct covariant quantization. $Q_{\mathrm{BRST}}$ is first rank. By using it to second quantize the system, we obtain a covariant off-shell unconstrained superfield action of the linearized $D=10$ type IIB supergravity. A corresponding procedure for the Green-Schwarz (GS) superstring is conjectured.

## 1. Introduction and motivation

In the last two years a lot of progress has been made in formulating Lorentz covariant first and second quantization procedures for the Ramond-Neveu-Schwarz superstrings [1,2] (for a long list of references see the book [3]) within the Batalin-Fradkin-Vilkovisky-Becchi-Rouet-Stora-Tyutin (BFV-BRST) approach [4,5]. In view of the importance of manifest space-time supersymmetry (anomaly cancellation, finiteness, vanishing cosmological constant etc.) for the superstrings [6,3], much interest was focussed also on the Green-Schwarz (GS) superstrings [7,8].

The original form of the latter exhibits, however, two major problems:

(i) It contains second class constraints which lead to highly complicated canonical Dirac brackets [8]. In particular, for the Brink-Schwarz superparticle (BS) [9,10] (the zero-mode approximation of the GS superstring) the ordinary superspace coordinates $x_\mu$, $\theta_\alpha$ do not commute and the initial connection with the geometry of the embedding superspace is lost.

(ii) The first class constraints – the local fermionic $\kappa$-symmetries – are functionally dependent when expressed covariantly. In the terminology of refs. [11,12], they form a reducible set. In ref. [13] it was pointed out that for the GS and BS systems, the BFV procedure [11,12] of treating correctly reducible constraints breaks Lorentz covariance if the level of reducibility is to remain finite.

Siegel's modification [14,15] of the GS superstring does not address problem (2). Moreover Siegel's superparticle [15] changes the physical content [16] of the BS

* Permanent address: Institute of Nuclear Research and Nuclear Energy, Boul. Lenin 72, 1784 Sofia, Bulgaria.

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superparticle. Consequently it does not describe anymore the zero-mode approximation of the GS superstring.

Thus, even the simpler problem of covariant (first and second) quantization of the Brink-Schwarz superparticle had not been solved previously (see ref. [3] for an update).

Recently [13] a new formalism was proposed which solves the problems (i) and (ii) for the $N=2$ superparticle in $D=10$ (the only relevant space-time dimension for the superstring generalization). The main ingredient was the introduction of additional pure gauge degrees of freedom – Lorentz spinor harmonic variables. These were crucial in order to obtain an irreducible set of constraints without destroying explicit Lorentz covariance and without altering the physical content of the model. However in this formalism $Q_{\mathrm{BRST}}$ was of rank two. This made it a very cumbersome tool to use in the eventual covariant second-quantization of the system. Also, ref. [13] did not succeed in extending off-shell the covariant reality condition for the superfield wave function. This was an obstacle for the off-shell unconstrained superspace formulation of the linearized $D=10$ type IIB supergravity (i.e. the second quantized $D=10$, $N=2$ BS superparticle).

The purpose of the present paper is threefold:

(a) We present a significantly simplified $D=10$ harmonic superspace for the $N=2$ BS superparticle. In particular the rank of the BRST charge is one. Also, the geometrical meaning of the new harmonics is much more transparent.

(b) Within the framework of BFV-BRST approach for the covariant second quantization of constrained systems [17] we find an off-shell unconstrained superfield action for the linearized $D=10$, $N=2$ type IIB supergravity.

(c) We elucidate the mechanism of cleaning the system from second class constraints without changing its physical content. In essence we recognize half of the second class constraints as the gauge fixing conditions for the other half which are thereby recognized as generators of new gauge transformations. By renouncing the gauge fixing these generators become first class constraints.

We conjecture that the same mechanism acts also in the case of the GS superstring. This conjecture is presently under study.

The plan of the paper is as follows.

In sect. 2, the classical $N=2$ BS superparticle is reformulated. We present the idea of expressing it as a system with first-class constraints only (at that stage these constraints are still dependent). The rigorous formulation of the idea is postponed until sect. 4 because it requires the formalism of sect. 3.

In sect. 3, a new type of $D=10$ harmonic superspace is introduced with both Lorentz-vector and Lorentz-spinor harmonic coordinates realizing the coset space $\mathrm{SO}(1,9)/(\mathrm{SO}(8)\times\mathrm{SO}(1,1))$.

These harmonics are used in sect. 4 to construct a new (physically equivalent) form of the action introduced in sect. 2. In this form, the whole set of constraints becomes irreducible. (We call it in short the harmonic superparticle action.)

Covariant first quantization *à la* Dirac of the $N=2$ harmonic superparticle is performed in sect. 5.

The BFV-BRST covariant second quantization is discussed in sect. 6. The first quantized BRST charge is shown to be of rank one and a covariant off-shell reality condition for the superfield wave function is found, enabling us to accomplish the task (b) declared above.

Appendices A and B are of technical nature and collect our spinor conventions and some useful algebraic properties of the harmonic expansions, respectively.

Appendix C contains the proof of the equivalence between the following two quantized *à la* Dirac systems:

(a) A system with $2n$ real second class constraints.

(b) A system with $n$ holomorphic first class constraints.

Throughout the paper we repeatedly check the equivalence of our covariant results with the known results about the point-limit of the light-cone gauge of the GS superstring (the type IIB supergravity). We regard this correspondence as our criterion of success.

## 2. Reformulation of $N=2$ BS superparticle

The standard form of the BS superparticle action in $N=2$, $D=10$ superspace $(x^{\mu}, \theta_{\alpha}^{A})$, $A=1,2$, reads:

$$S=\int \mathrm{d}\tau \left[ p_{\mu}\, \partial_{\tau} x^{\mu} + p_{\theta}^{A\alpha}\, \partial_{\tau}\theta_{\alpha}^{A} - H_{\mathrm{T}} \right], \tag{2.1}$$

$$H_{\mathrm{T}} = \lambda p^{2} + \psi_{\alpha}^{A} d^{A\alpha}. \tag{2.2}$$

In eq. (2.1), $\theta_{\alpha}^{A}$ are left-handed Majorana-Weyl (MW) spinors* and the fermionic constraints

$$d^{A\alpha} = -ip_{\theta}^{A\alpha} - \not{p}^{\alpha\beta}\theta_{\beta}^{A}, \tag{2.3}$$

$$\{d^{A\alpha}, d^{B\beta}\}_{\mathrm{PB}} = i2\delta^{AB}\not{p}^{\alpha\beta} \tag{2.4}$$

form a mixture of 16 first-class and 16 second-class constraints on the constraint surface $p^{2}=0$. Indeed, only half of the $\psi_{\alpha}^{A}$ are arbitrary Lagrange multipliers, the rest, being determined by the consistency of the dynamics generated by $H_{\mathrm{T}}$ with the whole set of constraints, see ref. [18].

* See appendix A for our spinor conventions.

For the present purpose it is convenient to use *the holomorphic representation* for the fermionic part [19] of the phase space:

$$\left(x^{\mu},\theta_{\alpha},\bar{\theta}_{\alpha};\,p_{\mu},\bar{p}_{\theta}^{\alpha},p_{\theta}^{\alpha}\right),$$

$$\theta_{\alpha}=\sqrt{\tfrac{1}{2}}\left(\theta_{\alpha}^{1}+i\theta_{\alpha}^{2}\right),$$

$$\bar{\theta}_{\alpha}=\sqrt{\tfrac{1}{2}}\left(\theta_{\alpha}^{1}-i\theta_{\alpha}^{2}\right),$$

$$p_{\theta}^{\alpha}=\sqrt{\tfrac{1}{2}}\left(p_{\theta}^{1\alpha}+ip_{\theta}^{2\alpha}\right),$$

$$\bar{p}_{\theta}^{\alpha}=\sqrt{\tfrac{1}{2}}\left(p_{\theta}^{1\alpha}-ip_{\theta}^{2\alpha}\right). \tag{2.5}$$

In this representation the action (2.1), the Lagrange multipliers and the constraints (2.3) read:

$$S=\int\mathrm{d}\tau\left[p_{\mu}\,\partial_{\tau}x^{\mu}+p_{\theta}^{\alpha}\,\partial_{\tau}\bar{\theta}_{\alpha}+\bar{p}_{\theta}^{\alpha}\,\partial_{\tau}\theta_{\alpha}-H_{\mathrm{T}}\right], \tag{2.6}$$

$$H_{\mathrm{T}}=\lambda p^{2}+\psi_{\alpha}\bar{d}^{\alpha}+\bar{\psi}_{\alpha}d^{\alpha}, \tag{2.7}$$

$$\psi_{\alpha}=\sqrt{\tfrac{1}{2}}\left(\psi_{\alpha}^{1}+i\psi_{\alpha}^{2}\right),$$

$$\bar{\psi}_{\alpha}=\sqrt{\tfrac{1}{2}}\left(\psi_{\alpha}^{1}-i\psi_{\alpha}^{2}\right),$$

$$d^{\alpha}=\sqrt{\tfrac{1}{2}}\left(d^{1\alpha}+id^{2\alpha}\right)=-ip_{\theta}^{\alpha}-\not{p}^{\alpha\beta}\theta_{\beta},$$

$$\bar{d}^{\alpha}=\sqrt{\tfrac{1}{2}}\left(d^{1\alpha}-id^{2\alpha}\right)=-i\bar{p}_{\theta}^{\alpha}-\not{p}^{\alpha\beta}\bar{\theta}_{\beta}. \tag{2.8}$$

The Poisson brackets (2.4) are rewritten in terms of the holomorphic constraints (2.8) as:

$$\{d^{\alpha},d^{\beta}\}_{\mathrm{PB}}=\{\bar{d}^{\alpha},\bar{d}^{\beta}\}_{\mathrm{PB}}=0, \tag{2.9}$$

$$\{d^{\alpha},\bar{d}^{\beta}\}_{\mathrm{PB}}=2i\not{p}^{\alpha\beta}. \tag{2.10}$$

In general it is always preferable, and sometimes essential to work with first-class constraints only. (The appearance of inverses of operators in the Dirac brackets might interfere with a local formulation of the theory [3].) Therefore, instead of the initial mixed set of 16 first- and 16 second-class constraints $d^{\alpha}$, $\bar{d}^{\alpha}$ (2.8) in (2.7) we want to take the following set of constraints [13]:

$$d^{\alpha}=0, \tag{2.11}$$

$$\not{p}\bar{d}^{\alpha}=0. \tag{2.12}$$

The set (2.11), (2.12) is first-class on the surface $p^2=0$:

$$\{d^\alpha, (\not{p}\bar{d})_\beta\}_{\mathrm{PB}} = -i2\delta^\alpha_\beta p^2 . \tag{2.13}$$

The set (2.11), (2.12) is not independent on the surface $p^2=0$ because the matrix $\not{p}$ has rank 8 on that surface. Therefore, eq. (2.12) represents only half of the 16 constraints $\bar{d}^\alpha$. The other half of the constraints $\bar{d}^\alpha$ (to be explicitly exposed in sect. 4) is now immediately recognized as a set of 8 (partial) antiholomorphic gauge fixing conditions for the complex gauge invariance generated by the 16 holomorphic first-class constraints $d^\alpha$. The general procedure to treat $2n$ real second-class constraints as $n$ holomorphic first class constraints is explained in the appendix C.

Disregarding the gauge fixing conditions, is of course not affecting the physical content of the theory. Thus, the classical action with full gauge invariance restored:

$$S_{\text{gauge inv}} = \int \mathrm{d}\tau \left[ p_\mu\, \partial_\tau x^\mu + p_\theta\, \partial_\tau \bar{\theta} + \bar{p}_\theta\, \partial_\tau \theta - H_{\text{gauge inv}} \right], \tag{2.14}$$

$$H_{\text{gauge inv}} = \lambda p^2 + \bar{\psi}_\alpha d^\alpha + \psi^\alpha (\not{p}\bar{d})_\alpha \tag{2.15}$$

is physically equivalent to the original BS $N=2$ action (2.1). (The technical details of the proof are given in the appendix C.)

The counting of the degrees of freedom also comes out right because the new set of 24 first-class constraints kills as many degrees of freedom as the initial set of 16 first-class plus 16 second-class constraints.

The details of this analysis will be given in the sect. 4.

We remark parenthetically that Siegel's modification would correspond to the elimination of a half of $d^\alpha$ in addition to the half of $\bar{d}^\alpha$. This would be overkill, because it would throw away the gauge generators together with the gauge fixings, leaving the system with less gauge invariances and therefore, with more physical degrees of freedom [16]. As emphasized above, the half of the $d^\alpha$'s (2.8) which used to be second-class in (2.10) becomes first-class once half of the $\bar{d}^\alpha$'s is eliminated to obtain (2.12).

After this preliminary description of the solution of the problem (i), one has still to discuss the solution of the problem (ii).

Indeed, the first-class part of the constraints $\bar{d}^\alpha$ is expressed through the 16-component spinor constraint $\not{p}\bar{d}$. However, in a proper quantization procedure one has to take into account that only 8 of these constraints are functionally independent. As already stressed in ref. [13] the correct way to do it without spoiling Lorentz-covariance is to introduce additional degrees of freedom – harmonic variables – carrying Lorentz-spinor indices.

In the next section, we introduce a new $D=10$, $N=2$ harmonic superspace which is substantially simpler than the one discussed in ref. [13] and it has more geometrical meaning.

## 3. $D=10$ harmonic superspace with spinor harmonics

The concept of harmonic superspaces was first proposed in ref. [20] as a fundamental ingredient of the unconstrained, off-shell superfield formulation of $N=2,3$ matter-, gauge- and supergravity theories in $D=4$. The main idea is to reduce by means of harmonic variables a global symmetry group G of the supersymmetric theory to an appropriate subgroup H, where the harmonics serve as "bridges" converting G-covariant indices into H-covariant ones, preserving at the same time the G-symmetry.

In $D=4$, one has the case $G=SU(N)$ (the automorphism group of $N$-extended supersymmetry) and $H=U(1)^{N-1}$ for $N=2,3$.

Subsequently, a different kind of $N=1$ harmonic superspace in $D=10$ – the light-cone harmonic superspace – was introduced [21] where $G=SO(1,9)$ is the Lorentz-group and $H=SO(8)\times SO(1,1)$ is the so called light-cone preserving subgroup. We would like to warn the reader that this name is somewhat misleading: the group H is local and internal, and so is the "light cone" which it preserves. There is no tampering with the Lorentz-invariance.

The corresponding harmonic variables $(u_\mu^{\pm}, u_\mu^{i})$ were defined as follows [21]:

$$
\begin{aligned}
u_\mu^{\pm} u^{\pm\mu} &= 0, \\
u_\mu^{+} u^{-\mu} &= -1, \\
u_\mu^{i} u^{j\mu} &= \delta^{ij}, \\
u_\mu^{i} u^{\pm\mu} &= 0,
\end{aligned}
\tag{3.1}
$$

where $H=SO(8)\times SO(1,1)$ acts as local rotations on the internal SO(8) indices $i, j=1\ldots 8$, and SO(1,1)-indices $\pm$.

Yet another type of harmonic superspace in $D=10$ [13] contains the harmonics

$$
\left(v_\alpha^{a,\dot a}, \tilde v_{a,\dot a}^{\beta}\right)
\tag{3.2}
$$

which are defined to satisfy the constraints:

$$
v_\alpha^{a}\tilde v_a^{\beta} + v_\alpha^{\dot a}\tilde v_{\dot a}^{\beta} = \delta_\alpha^{\beta}.
\tag{3.3}
$$

These harmonics were introduced as "bridges" converting $D=10$ left- (right-) handed MW spinor indices $\alpha, \dot\alpha$ into SO(8) $(s),(c)$ spinor indices $a, \dot a$.

As explained in ref. [13] the use of harmonics carrying $D=10$ MW indices is inevitable in order to express in a functionally independent way the first-class part of the constraint $\bar d^{\alpha}$.

We now propose a new $D=10$ harmonic superspace which combines the nice features of both [21] and [13]. It consists of the following objects: $v_\alpha^{\pm 1/2}$ – two

$D=10$ (left-handed) MW spinors and $u^a_\mu$ – eight ($a=1,\dots,8$), $D=10$ Lorentz vectors satisfying the constraints:

$$\begin{aligned}\left[v_\alpha^{+1/2}(\sigma^\mu)^{\alpha\beta}v_\beta^{+1/2}\right]\left[v_\gamma^{-1/2}(\sigma_\mu)^{\gamma\delta}v_\delta^{-1/2}\right] &= -1\,,\\ \left[v_\alpha^{\pm1/2}(\sigma^\mu)^{\alpha\beta}v_\beta^{\pm1/2}\right]u^a_\mu &= 0\,,\\ u^a_\mu u^\mu_b &= \delta^a_b\,.\end{aligned}\tag{3.4}$$

Under the local rotations of the internal local subgroup $\mathrm{H}=\mathrm{SO}(8)\times\mathrm{SO}(1,1)$, $u^a_\mu$ transform as SO(8) ($s$)-spinors whereas $v_\alpha^{\pm1/2}$ carry $\pm\frac{1}{2}$ charge under SO(1,1).

The geometric meaning of (3.4) becomes immediately clear when one recalls the celebrated $D=10$ Fierz identity (cf. e.g. ref. [3]):

$$(\sigma_\mu)^{\alpha\beta}(\sigma^\mu)^{\gamma\delta}+(\sigma_\mu)^{\beta\gamma}(\sigma^\mu)^{\alpha\delta}+(\sigma_\mu)^{\gamma\alpha}(\sigma^\mu)^{\beta\delta}=0\,.\tag{3.5}$$

Indeed, the following composite vectors

$$u^\pm_\mu=v_\alpha^{\pm1/2}(\sigma_\mu)^{\alpha\beta}v_\beta^{\pm1/2}\tag{3.6}$$

are identically light-like because of (3.5). Using the set of $u^\pm_\mu$ (3.6) together with $u^a_\mu$ from (3.4) one obtains a realization of the coset-space $\mathrm{SO}(1,9)/(\mathrm{SO}(8)\times\mathrm{SO}(1,1))$ as in (3.1). The only difference is that SO(8) is taken in the ($s$)-spinor representation instead of the vector one.

Henceforth, we shall use the shorthand notations:

$$\begin{aligned}&u^\pm_\mu \qquad \text{as in (3.6)}\,,\\ &\sigma^a\equiv\sigma^\mu u^a_\mu\,,\\ &\sigma^\pm\equiv\sigma^\mu\left(v^{\pm1/2}\sigma_\mu v^{\pm1/2}\right).\end{aligned}\tag{3.7}$$

The following harmonic differential operators (which preserve the harmonic constraints (3.4)) will play an important role in the sequel:

$$\begin{aligned}D^{ab}&=u^a_\mu\frac{\partial}{\partial u_{\mu b}}-u^b_\mu\frac{\partial}{\partial u_{\mu a}}\,,\\ D^{+a}&=u^+_\mu\frac{\partial}{\partial u_{\mu a}}+\tfrac{1}{2}v^{-1/2}\sigma^+\sigma^a\frac{\partial}{\partial v^{-1/2}}\,,\\ D^{-+}&=\tfrac{1}{2}v_\alpha^{+1/2}\frac{\partial}{\partial v_\alpha^{+1/2}}-\tfrac{1}{2}v_\alpha^{-1/2}\frac{\partial}{\partial v_\alpha^{-1/2}}\,.\end{aligned}\tag{3.8}$$

$D^{ab}$, $D^{-+}$ and $D^{+a}$ form a closed algebra and the first two of them are easily recognized as generators of SO(8) × SO(1,1) where the subgroup SO(8) is taken in the $(s)$-spinor representation:

$$\begin{aligned}
&[D^{ab}, D^{cd}] = C^{bc}D^{ad} - C^{ac}D^{bd} + C^{ad}D^{bc} - C^{bd}D^{ac}, \\
&[D^{-+}, D^{ab}] = 0, \\
&[D^{ab}, D^{+c}] = C^{bc}D^{+a} - C^{ac}D^{+b}, \\
&[D^{-+}, D^{+a}] = D^{+a}, \\
&[D^{+a}, D^{+b}] = 0.
\end{aligned} \tag{3.9}$$

($C^{ab}$ denotes the $D=8$ chiral charge conjugation matrix.)

As in all previous types of harmonic superspaces [21,20], the general harmonic superfield is defined by the following harmonic expansion:

$$\begin{aligned}
\Phi(x,\theta,\bar{\theta},u,v) = \sum_{n,m=0}^{\infty} & \left[u^{a_1}_{\mu_1} \dots u^{a_n}_{\mu_n}\right]_{\text{singlet part in } (a_1 \dots a_n)} \\
& \times v^{+1/2}_{\alpha_1} \dots v^{+1/2}_{\alpha_m} v^{-1/2}_{\alpha_{m+1}} \dots v^{-1/2}_{\alpha_{2m}} \Phi^{\mu_1 \dots \mu_n \alpha_1 \dots \alpha_{2m}}(x,\theta,\bar{\theta}),
\end{aligned} \tag{3.10}$$

i.e. the coefficients are ordinary superfields which do not carry any indices of the internal subgroup SO(8) × SO(1,1). This property will have important implications for the substantial simplification of the superparticle constraint algebra in the following sections.

## 4. $N=2$ harmonic superparticle

Equipped with the harmonic formalism of the preceding section, we can now explicitly solve the problem posed in sect. 2: the Lorentz-covariant separation of the first-class and second-class constraints from the constraints of the action (2.6)–(2.7). The solution is provided by the following decomposition of each of the 16-component $D=10$ spinors $d^\alpha, \bar{d}^\alpha$ (2.8) into direct sums of two 8-component SO(8) $(s)$-spinors:

$$\begin{aligned}
d^\alpha &= 2(\sigma^a v^{+1/2})^\alpha d_a^{-1/2} + (p^+)^{-1}(\not{p}\sigma^+\sigma^a v^{-1/2})^\alpha g_a^{+1/2} \\
\bar{d}^\alpha &= (p^+)^{-1}(\sigma^a v^{+1/2})^\alpha (\bar{d})_a^{+1/2} + (\not{p}\sigma^+\sigma^a v^{-1/2})^\alpha \bar{g}_a^{-1/2},
\end{aligned} \tag{4.1}$$

where

$$p^+ \equiv v^{+1/2}\not{p}v^{+1/2}$$

and

$$d^{-1/2\,a} = (2p^+)^{-1}\left(v^{+1/2}\sigma^a \not{p} d\right),$$

$$\bar{d}^{+1/2\,a} = \left(v^{+1/2}\sigma^a \not{p}\bar{d}\right), \tag{4.2}$$

$$g^{+1/2\,a} = \tfrac{1}{2}\left(v^{-1/2}\sigma^a\sigma^+ d\right), \tag{4.3}$$

$$\bar{g}^{-1/2\,a} = (2p^+)^{-1}\left(v^{-1/2}\sigma^a\sigma^+\bar{d}\right). \tag{4.4}$$

The canonical Poisson brackets (2.9), (2.10) are now written as:

$$\{d^{-1/2\,a}, \bar{d}^{+1/2\,b}\}_{PB} = -iC^{ab}p^2, \tag{4.5}$$

$$\{g^{+1/2\,a}, \bar{g}^{-1/2\,b}\}_{PB} = iC^{ab}, \tag{4.6}$$

$$\text{rest PB} = 0.$$

Let us emphasize that the SO(8) and $\pm$ indices are internal and, therefore, all the expressions are explicitly Lorentz-covariant. In particular, the formulation of the second class constraints (4.3), (4.4) in a covariant form is a long awaited result [22, 23, 3] for the BS superparticle.

From (4.6) it is clear that we can take $d^\alpha$ (or, equivalently, $d^{-1/2\,a}$ and $g^{+1/2\,a}$) and $\bar{d}^{+1/2\,a}$ as covariant functionally independent first-class constraints of the $N=2$ superparticle whereas $\bar{g}^{-1/2\,a}$ is interpreted as a gauge fixing condition for the first-class constraint $g^{+1/2\,a}$.

Thus we arrive at the following form of the action for the $D=10$, $N=2$ BS superparticle:

$$S_{\text{harmonic}} = \int \mathrm{d}\tau \Big[ p_\mu\, \partial_\tau x^\mu + p_\theta^\alpha\, \partial_\tau \theta_\alpha + \bar{p}_\theta^\alpha\, \partial_\tau \bar{\theta}_\alpha + p_{u_\mu}^a\, \partial_\tau u_a^\mu$$

$$+ p_v^{\mp 1/2\,\alpha}\, \partial_\tau v_\alpha^{\pm 1/2} - H_{\text{harmonic}} \Big], \tag{4.7}$$

$$H_{\text{harmonic}} = \lambda p^2 + \bar{\psi}_\alpha d^\alpha + \psi_a^-(\bar{d})^{+1/2\,a} + \Lambda_{ab} d^{ab} + \Lambda^{+-} d^{-+} + \Lambda_a^- d^{+a}. \tag{4.8}$$

The constraints $d^{ab}$, $d^{+-}$, $d^{+a}$ in (4.8) denote the classical counterparts of $D^{ab}$, $D^{-+}$, $D^{+a}$ in (3.8), respectively.

This action contains only first class, functionally independent, constraints and is therefore suitable for the BFV-BRST covariant procedure without need for introducing new "constraints for the constraints". We will see in the sequel of this article that, upon quantization, it gives correctly the point-limit of the GS superstring, i.e. the type IIB supergravity in $D=10$.

Because of the kinematical constraints (3.4) on the variables $u^a_\mu, v^{\pm 1/2}_\alpha$ defining our harmonic superspace, their conjugate momenta are similarly kinematically constrained:

$$p_u^{\mu(a}u_\mu^{b)} = 0\,,$$

$$p^a_{u_\mu}\left(v^{\pm 1/2}\sigma^\mu v^{\pm 1/2}\right) = 0\,,$$

$$v_\alpha^{+1/2}p_v^{-1/2\,\alpha} + v_\alpha^{-1/2}p_v^{+1/2\,\alpha} = 0\,. \tag{4.9}$$

The constraints (3.4) and (4.9) may be equivalently regarded as a system of conjugated second-class constraints and thus all subsequent Poisson-bracket relations are in fact Dirac-bracket relations on the surface defined by (3.4) and (4.9).

The spinor constraint:

$$v^{+1/2}\sigma^a \not{p}\bar{d} \equiv \bar{d}^{+1/2\,a} \tag{4.10}$$

is precisely the functionally independent Lorentz-covariant first-class part of the constraint $\bar{d}^\alpha$.

The algebra of constraints in (4.8) reads (only the nonzero PB's are listed):

$$\{d^\alpha, \bar{d}^{+1/2\,a}\}_{\mathrm{PB}} = -i2p^2 v_\beta^{+1/2}(\sigma^a)^{\beta\alpha}\,,$$

$$\{d^{-+}, \bar{d}^{+1/2\,a}\}_{\mathrm{PB}} = \tfrac{1}{2}\bar{d}^{+1/2\,a}\,, \tag{4.11}$$

$$\{d^{ab}, \bar{d}^{+1/2\,c}\}_{\mathrm{PB}} = C^{bc}\bar{d}^{+1/2\,a} - C^{ac}\bar{d}^{+1/2\,b}\,, \tag{4.12}$$

$d^{ab}, d^{+-}, d^{+a}$ commute among themselves as in (3.9).

Any smooth function on the phase space of the superparticle system (4.7) including classical observables, is defined by the following harmonic expansions:

$$\begin{aligned} F(z, u, v;\, p_z, p_u, p_v) = \sum \left[u^{a_1}_{\mu_1}\dots u^{a_n}_{\mu_n} p^{a_{n+1}}_{u\mu_{n+1}}\dots p^{a_{n+k}}_{u\mu_{n+k}}\right]_{\text{singlet part in }(a_1,\dots,a_{n+k})} \\ \times v^{+1/2}_{\alpha_1}\dots v^{+1/2}_{\alpha_m} v^{-1/2}_{\alpha_{m+1}}\dots v^{-1/2}_{\alpha_{m+r}} p_v^{\beta_1,+1/2}\dots \\ p_v^{\beta_l,+1/2} p_v^{\beta_{l+1},-1/2}\dots p_v^{\beta_{m+l-r},-1/2} \\ \times F^{\mu_1\dots\mu_{n+k},\alpha_1\dots\alpha_{m+r}}_{\beta_1\dots\beta_{m+l-r}}(z, p_z)\,, \end{aligned} \tag{4.13}$$

where, of course $u^a_\mu, v^{\pm 1/2}_\alpha, p^a_{u_\mu}, p_v^{\alpha,\pm 1/2}$ are constrained on the surface (3.4), (4.9). Let us emphasize, that the coefficient functions in the harmonic expansion (4.13) do not carry any $SO(8)\times SO(1,1)$ internal indices, precisely as in the case of the harmonic superfields (3.10).

Physical observables must have vanishing Poisson brackets with all constraints in (4.8). Starting with the relations:

$$\{F, d^{ab}\}_{\mathrm{PB}} = \{F, d^{-+}\}_{\mathrm{PB}} = \{F, d^{+a}\}_{\mathrm{PB}} = 0\,, \tag{4.14}$$

where $F$ is of the form (4.13), one can show that such functions do not depend on the harmonic coordinates and their conjugate momenta. Thus, the harmonics are in fact pure gauge degrees of freedom in the harmonic superparticle system (4.7). Thereby, the harmonic superparticle system (4.7) is physically equivalent to the usual $N=2$ BS superparticle. We shall discuss the equivalence proof in more detail directly on the first-quantized level in the next section.

## 5. Covariant first quantization

According to (4.8), the covariant first-quantized Dirac-constrained equations for the harmonic superparticle read:

$$p^2\Phi=0, \tag{5.1}$$

$$D^{\alpha}\Phi=0, \tag{5.2}$$

$$\overline{D}^{+1/2\,a}\Phi\equiv v^{+1/2}\sigma^{a}\not{p}\overline{D}\Phi=0, \tag{5.3}$$

$$D^{ab}\Phi=0, \tag{5.4}$$

$$D^{-+}\Phi=0, \tag{5.5}$$

$$D^{+a}\Phi=0, \tag{5.6}$$

where now

$$D^{\alpha}=\frac{\partial}{\partial\overline{\theta}_{\alpha}}-\not{p}^{\alpha\beta}\theta_{\beta},$$

$$\overline{D}^{\alpha}=\frac{\partial}{\partial\theta_{\alpha}}-\not{p}^{\alpha\beta}\overline{\theta}_{\beta},$$

and $D^{ab}, D^{-+}, D^{+a}$ are as in (3.8). $\Phi=\Phi(z,u,v)$ is (3.10) taken in the $p_\mu$ momentum space, i.e., $z=(p_\mu,\theta,\overline{\theta})$.

We want to analyse the physical content of the system characterized by (5.1)–(5.6). The main effort will be invested now in the study of the effects of the conditions (5.4)–(5.6) on the form of the superfield (3.10). In spite of the technicalities involved in this study, its outcome will be very simple: the variables $u, v$ are pure gauge degrees of freedom which are eliminated on-shell by the constraints (5.4)–(5.6). The reader not interested in the details, may as well jump to the conclusion of this analysis: equation (5.11).

Accounting the harmonic constraints (3.4), the general solution to (5.4), (5.5) is of the form (see appendix B):

$$\Phi(z,u,v)=\sum_{n=0}^{\infty}u^{+}_{[\mu]_1}\cdots u^{+}_{[\mu]_n}u^{-}_{[\nu]_1}\cdots u^{-}_{[\nu]_n}\times\left\{\Phi_n^{([\mu]_1\cdots[\mu]_n)([\nu]_1\cdots[\nu]_n)}(z)+r_{[\lambda]}Y_n^{([\mu]_1\cdots[\mu]_n)([\nu]_1\cdots[\nu]_n)[\lambda]}(z)\right\} \tag{5.7}$$

with the notations:

$$u^{\pm}_{[\mu]} = v^{\pm 1/2}\sigma_{[\mu]}v^{\pm 1/2},$$

$$r_{[\mu]} = v^{+1/2}\sigma_{[\mu]}v^{-1/2},$$

where $\sigma_{[\mu]}$ denote antisymmetrized products of one, three and five $D=10$ $\sigma$-matrices (see appendix A). Note that $u^a_\mu$ do not enter in (5.7) since any SO(8)-invariant combination of $u^a_\mu$'s can be expressed solely in terms of $u^\pm_\mu$ on the surface (3.4):

$$u^a_\mu u_{a\nu} = \eta_{\mu\nu} + u^+_\mu u^-_\nu + u^-_\mu u^+_\nu ,$$

$$\varepsilon_{a_1\dots a_8}u^{a_1}_{\mu_1}\dots u^{a_8}_{\mu_8} = -\varepsilon^{\nu_1\cdots\nu_{10}}u^+_{\nu_9}u^-_{\nu_{10}}\prod_{k=1}^{8}\left[\eta\mu_k\nu_k + u^+_{\mu_k}u^-_{\nu_k} + u^-_{\mu_k}u^+_{\nu_k}\right]. \tag{5.8}$$

Further we have used in (5.7) the fact that a product of two $v^{\pm 1/2}_\alpha$'s can, equivalently, be reexpressed (cf. eq. (A.4)) in terms of $u^\pm_{[\mu]}$ and/or $r_{[\lambda]}$ as:

$$-v^{\pm 1/2}_\alpha v^{\pm 1/2}_\beta = \frac{1}{16}\sigma^\mu_{\alpha\beta}u^\pm_\mu + \frac{1}{32(5!)}\sigma^{\mu_1\cdots\mu_5}_{\alpha\beta}u^\pm_{\mu_1\dots\mu_5},$$

$$-v^{+1/2}_\alpha v^{-1/2}_\beta = \frac{1}{16}\sigma^\mu_{\alpha\beta}r_\mu + \frac{1}{16(3!)}\sigma^{\mu_1\cdots\mu_3}_{\alpha\beta}r_{\mu_1\dots\mu_3} + \frac{1}{32(5!)}\sigma^{\mu_1\cdots\mu_5}_{\alpha\beta}r_{\mu_1\dots\mu_5}. \tag{5.9}$$

Also, in appendix B it is shown that, contractions among Lorentz indices in products $u^\pm_{[\mu]}u^\pm_{[\nu]}$ or $u^\pm_{[\mu]}r_{[\lambda]}$ are either 0, or are expressed in terms of $u^\pm_{[\mu]}u^\pm_{[\nu]}$ or $u^\pm_{[\mu]}r_{[\lambda]}$ with shorter index sets $[\mu],[\nu],[\lambda]$. The proof uses the Fierz identity and the harmonic constraints (3.4).

Therefore, the only allowed trace parts in the tensor coefficient superfields in (5.7) are of the form:

$$Y^{(\mu[\mu]_2\dots[\mu]_n)([\nu]_1\dots[\nu]_n)[\lambda_1\dots\lambda_5]} \sim \eta^{\mu\lambda_1}\tilde{Y}^{([\mu]_2\dots[\mu]_n)([\nu]_1\dots[\nu]_n)\lambda_2\dots\lambda_5},$$

$$Y^{([\mu]_1\dots[\mu]_n)(\nu[\nu]_2\dots[\nu]_n)[\lambda_1\dots\lambda_5]} \sim \eta^{\nu\lambda_1}\tilde{Y}^{([\mu]_1\dots[\mu]_n)([\nu]_2\dots[\nu]_n)\lambda_2\dots\lambda_5},$$

$$\Phi^{(\mu[\mu]_2\dots[\mu]_n)([\nu_1\dots\nu_5][\nu]_2\dots[\nu]_n)} \sim \eta^{\mu\nu_1}\tilde{\Phi}^{([\mu]_2\dots[\mu]_n)(\nu_2\dots\nu_5[\nu]_2\dots[\nu]_n)},$$

$$\Phi^{([\mu_1\dots\mu_5][\mu]_2\dots[\mu]_n)(\nu[\nu]_2\dots[\nu]_n)} \sim \eta^{\mu_1\nu}\tilde{\Phi}^{(\mu_2\dots\mu_5[\mu]_2\dots[\mu]_n)([\nu]_2\dots[\nu]_n)},$$

$$\Phi^{([\mu_1\dots\mu_5][\mu]_2\dots[\mu]_n)([\nu_1\dots\nu_5][\nu]_2\dots[\nu]_n)} \sim \eta^{\mu_1\nu_1}\tilde{\Phi}^{(\mu_2\dots\mu_5[\mu]_2\dots[\mu]_n)(\nu_2\dots\nu_5[\nu]_2\dots[\nu]_n)}, \tag{5.10}$$

and similarly for $Y^{([\mu]_1\dots)([\nu]_1\dots)[\lambda]}$. With the property (5.10) of the expansion (5.7) and accounting for the relations (B.2) it is now straightforward to show that (5.6) implies:

$$\Phi_n^{([\mu]_1\ldots)([\nu]_1\ldots)}(z)=0, \qquad n\geqslant 1,$$

$$Y_n^{([\mu]_1\ldots)([\nu]_1\ldots)[\lambda]}(z)=0, \qquad n\geqslant 0. \tag{5.11}$$

Consequently, the only surviving coefficient superfield in (5.7) is $\Phi_0(z)$.

Since $v_\alpha^{+1/2}$, $u_\mu^a$ are arbitrary coordinates of the harmonic coset space (3.4), then (5.3) implies that, in fact, $\Phi=\Phi_0(z)$ satisfies the equation

$$(\not{p}\bar{D})_\alpha \Phi_0(z)=0. \tag{5.12}$$

In conclusion we find that the set of constraint equations (5.1)–(5.3) for the solution $\Phi=\Phi_0(z)$ of (5.4)–(5.6) precisely coincides with the set of Dirac constraint equations for the system (2.14). Therefore the systems (4.7) and (2.14) are physically equivalent. This verifies (at the first quantized level) the equivalence between the standard BS and the harmonic superparticle systems in $D=10$, $N=2$. This check is by no means trivial because, in previous situations, trials to improve the constraint structure of the system by the introduction of new "gauge" variables ended in significant changes of the physical content of the theory [15, 21].

The contact with the light cone formulation can be achieved by solving (5.12) in a particular Lorentz frame. The result will be that in the light-cone gauge, the harmonic superparticle produces the type IIB supergravity. Later, we will find that, in fact, our formulation allows us to obtain the linearized type IIB supergravity also covariantly.

Let us now introduce the standard light-cone projectors (we use the hat to distinguish these quantities from the Lorentz-invariant (3.7) used throughout the rest of the paper):

$$\tfrac{1}{2}\hat{\sigma}^{\pm}\hat{\sigma}^{\mp},$$

$$\hat{\sigma}^{\pm}=\sqrt{\tfrac{1}{2}}\,(\sigma^0\pm\sigma^9). \tag{5.13}$$

Returning to the $x$ representation, the solution of (5.1), (5.2), (5.12) then reads:

$$\Phi=\mathrm{e}^{\bar{\theta}\not{p}\theta}\hat{\Phi}_0(x,\hat{\theta}),$$

$$p^2\hat{\Phi}_0=0, \tag{5.14}$$

where $\hat{\theta}$ is an 8-component SO(8) ($s$)-spinor obtained from $\theta_\alpha$ by the light-cone projection:

$$\hat{\theta}=\tfrac{1}{2}\hat{\sigma}^{-}\hat{\sigma}^{+}(\hat{p}^i\hat{\sigma}^i-\hat{p}^{+}\hat{\sigma}^{-})\theta,$$

$$\hat{p}^{\pm}=\sqrt{\tfrac{1}{2}}\,(p^0\pm p^9). \tag{5.15}$$

After imposing the reality condition [24] (* denotes usual complex conjugation):

$$\left[\hat{\Phi}_0(x,\hat{\theta})\right]^* = (\hat{p}^+)^4 \int \mathrm{d}^8\hat{\theta}' \exp\left\{ i\frac{\hat{\theta}^*\hat{\theta}'}{\hat{p}^+} \right\} \hat{\Phi}_0(x,\hat{\theta}') \tag{5.16}$$

the light-cone superfield $\hat{\Phi}_0(x,\hat{\theta})$ provides a description of the linearized $D=10$ type IIB supergravity [24]. Its off-shell light-cone action reads

$$S_{\text{light-cone}} = \tfrac{1}{4}\int \mathrm{d}^{10}x\, \mathrm{d}^8\hat{\theta}\, \hat{\Phi}_0(x,\hat{\theta})\, p^2 \hat{\Phi}_0(x,\hat{\theta}). \tag{5.17}$$

We expand $\hat{\Phi}_0$ in terms of its component fields:

$$\hat{\Phi}_0(x,\hat{\theta}) = \sum_{k=0}^{8} \frac{1}{k!} \hat{\theta}^{a_1} \dots \hat{\theta}^{a_k} \hat{\phi}_{a_1 \dots a_k}(x). \tag{5.18}$$

Eqs. (5.16) and (5.17) read for the components:

$$\hat{\phi}^*_{a_1 \dots a_k}(x) = -\frac{1}{(8-k)!}(i)^k (\hat{p}^+)^{4-k} \varepsilon_{a_1 \dots a_k b_1 \dots b_{8-k}} \hat{\phi}^{b_1 \dots b_{8-k}}(x), \tag{5.19}$$

$$\begin{aligned} S_{\text{light-cone}} = & -\sum_{l=0}^{2} \frac{1}{(2l)!} \int \mathrm{d}^{10}x \left[ (\hat{p}^+)^{l-2} \hat{\phi}^{*a_1 \dots a_{2l}}(x) \right] p^2 \left[ (\hat{p}^+)^{l-2} \hat{\phi}_{a_1 \dots a_{2l}}(x) \right] \\ & -\sum_{l=0}^{1} \frac{1}{(2l+1)!} \int \mathrm{d}^{10}x \left[ (\hat{p}^+)^{l-2} \hat{\phi}^{*a_1 \dots a_{2l+1}}(x) \right] \\ & \times \frac{ip^2}{p^+} \left[ (\hat{p}^+)^{l-2} \hat{\phi}_{a_1 \dots a_{2l+1}}(x) \right]. \end{aligned} \tag{5.20}$$

These are precisely the light-cone formulae of ref. [24]. We regard their reproduction by our formalism as a proof of the correctness of our covariant action (4.7).

We made this noncovariant digression just to establish the equivalence on-shell of our covariant formalism with the standard light-cone gauge results [24]. However, within our formalism we are not limited to the noncovariant solution. In fact, the system (5.1)–(5.6) for the harmonic superparticle may be solved in a manifestly Lorentz covariant form. Its solution is made obvious by:

(i) Performing a unitary transformation on $\Phi$:

$$\Phi(z,u,v) \to \mathrm{e}^{\bar{\theta}\not{p}\theta} \Phi(z,u,v). \tag{5.21}$$

(ii) Performing the change of variables $\theta_\alpha \to \theta^{\pm 1/2\, a}$ where

$$\begin{aligned} \theta^{+1/2\, a} &= -\tfrac{1}{2}\left(v^{-1/2}\sigma^+\sigma^a \not{p}\theta\right), \\ \theta^{-1/2\, a} &= (p^+)^{-1}\left(v^{+1/2}\sigma^a\theta\right). \end{aligned} \tag{5.22}$$

Under the above transformations the first-quantized constraints are rewritten in the form:

$$D^{\alpha}=\frac{\partial}{\partial\bar{\theta}_{\alpha}},$$

$$\bar{D}_{a}^{+1/2}=\frac{-i\partial}{\partial\theta^{-1/2,a}}+2\left(v^{+1/2}\sigma_{a}\bar{\theta}\right)p^{2},$$

$$\tilde{D}^{ab}=u_{\mu}^{a}\frac{\partial}{\partial u_{\mu b}}-u_{\mu}^{b}\frac{\partial}{\partial u_{\mu a}}+\sum_{\pm}\left[\theta^{\pm1/2a}\frac{\partial}{\partial\theta_{b}^{\pm1/2}}-\theta^{\pm1/2b}\frac{\partial}{\partial\theta_{a}^{\pm1/2}}\right],$$

$$\tilde{D}^{-+}=\tfrac{1}{2}\left(v_{\alpha}^{+1/2}\frac{\partial}{\partial v_{\alpha}^{+1/2}}-v_{\alpha}^{-1/2}\frac{\partial}{\partial v_{\alpha}^{-1/2}}\right)+\tfrac{1}{2}\left(\theta^{+1/2a}\frac{\partial}{\partial\theta^{+1/2a}}-\theta^{-1/2a}\frac{\partial}{\partial\theta^{-1/2a}}\right)\tag{5.23}$$

and $D^{+a}$ remains unchanged as in (3.8) because of the property:

$$D^{+a}\theta^{\pm1/2b}=0.\tag{5.24}$$

Inserting (5.23) into eqs. (5.1)–(5.6) we get the Lorentz-covariant solution:

$$\Phi(z,u,v)=\sum_{k=0}^{8}\frac{1}{k!}\theta^{+1/2a_1}\dots\theta^{+1/2a_k}\phi_{a_1\dots a_k}^{-k/2}(p,u,v),\tag{5.25}$$

$$D^{+a}\phi_{a_1\dots a_k}^{-k/2}(p,u,v)=0,$$

$$p^2\phi_{a_1\dots a_k}^{-k/2}(p,u,v)=0.\tag{5.26}$$

Comparing (5.25) with (5.18) we see that the fields $\phi_{a_1\dots a_k}^{-k/2}(p,u,v)$ satisfying (5.26), when considered in the $x$ representation, are precisely the Lorentz-covariant form of the GS light-cone component fields $\hat{\phi}_{a_1\dots a_k}(x)$ in (5.18).

Now, accounting for (5.23) and (5.24) we are able to write down an *off-shell* Lorentz covariant reality condition for the superfield wave function consistent with all the constraint equations (5.1)–(5.6):

$$\begin{aligned}&\left[\Phi\left(-p,\theta'^{+1/2a},(\theta'^{-1/2a})^{*},\theta,u,v\right)\right]^{*}\\&\quad=(p^{+})^{4}\int d^{8}\phi^{+1/2a}\exp\left\{i(p^{+})^{-1}(\theta^{+1/2a})^{*}\phi a^{+1/2}\right\}\\&\quad\times\Phi\left(p,\phi^{+1/2a},\theta'^{-1/2a},\bar{\theta},u,v\right).\end{aligned}\tag{5.27}$$

On-shell, eq. (5.27) reduces to (5.16).

Hereby, the quantized $N=2$ harmonic superparticle provides a covariant on-shell description of the linearized $D=10$ IIB supergravity multiplet. The off-shell (i.e. action-principle) description will be given in the next section.

## 6. BFV-BRST quantization

According to the general theory [4], the BRST charge $Q_{\mathrm{BRST}}$ may contain higher order ghost terms if the canonical PB relations among the first-class constraints involve nontrivial first-order structure functions (i.e. structure "constants" of the algebra of constraints which depend on the canonical variables).

The latter situation indeed arises in one of the PB relations for the harmonic superparticle constraints (the first line in eq. (4.11)).

However, one can check straightforwardly, e.g. by using the explicit formulas of ref. [4], that in the present case, the second order structure functions and, therefore, all higher structure functions identically vanish. This is due to the fact that the nontrivial structure function in (4.11) does not depend on the canonical momenta of the harmonic variables.

Thus, $Q_{\mathrm{BRST}}$ of the $N=2$ harmonic superparticle (4.7) is first rank*:

$$Q_{\mathrm{BRST}} = Q_0 + Q_{\mathrm{harmonic}}\,, \tag{6.1}$$

$$\begin{aligned} Q_0 = {} & \alpha^{-1}\left[cp^2 + i\left(c\frac{\partial}{\partial c} - \frac{1}{2}\right)\frac{\partial}{\partial \tilde{c}}\right] + i\frac{\partial}{\partial \alpha}\frac{\partial}{\partial \tilde{c}} + \bar{\chi}_\beta D^\beta + \chi_a^{-1/2}\overline{D}^{+1/2\,a} \\ & - \chi a^{-1/2}\left(v^{+1/2}\sigma^a\bar{\chi}\right)\alpha\frac{\partial}{\partial c} - \frac{\partial}{\partial \bar{\psi}_\beta}\frac{\partial}{\partial \tilde{\chi}^\beta} - \frac{\partial}{\partial \psi_a^{-1/2}}\frac{\partial}{\partial \tilde{\bar{\chi}}^{+1/2\,a}}\,, \end{aligned} \tag{6.2}$$

where $\alpha \equiv \sqrt{2\lambda}$,

$$\begin{aligned} Q_{\mathrm{harmonic}} = {} & i\eta_{ab}\left[D^{ab} + \chi^{-1/2\,a}\frac{\partial}{\partial \chi_b^{-1/2}} - \chi^{-1/2\,b}\frac{\partial}{\partial \chi_a^{-1/2}} + \eta^{-a}\frac{\partial}{\partial \eta_b^-} - \eta^{-b}\frac{\partial}{\partial \eta_a^-}\right. \\ & \left. + \eta_d^a\frac{\partial}{\partial \eta_{bd}} - \eta_d^b\frac{\partial}{\partial \eta_{ad}}\right] \\ & + i\eta\left[D^{-+} - \tfrac{1}{2}\chi_a^{-1/2}\frac{\partial}{\partial \chi_a^{-1/2}} - \eta_a^-\frac{\partial}{\partial \eta_a^-}\right] \\ & + i\eta_a^- D^{+a} + i\frac{\partial}{\partial \Lambda_{ab}}\frac{\partial}{\partial \tilde{\eta}^{ab}} + i\frac{\partial}{\partial \Lambda^{+-}}\frac{\partial}{\partial \tilde{\eta}} + i\frac{\partial}{\partial \Lambda^{-a}}\frac{\partial}{\partial \tilde{\eta}_a^+}\,. \end{aligned} \tag{6.3}$$

* This form of $Q_{\mathrm{BRST}}$ is obtained from the standard form by using the unitary transformation [25]: $Q_{\mathrm{BRST}} \to UQ_{\mathrm{BRST}}U^{-1}$, $\ln U = -(\ln\alpha)(c\,\partial/\partial c + \tilde{c}\,\partial/\partial\tilde{c} - 1)$, $\alpha \equiv \sqrt{2\lambda}$.

The variables appearing in the above expression of the BRST charge are organized as follows:

$$\begin{bmatrix} \text{Lagrange multiplier} & \text{ghost} & \text{antighost} & \text{of the constraint} \\ \alpha & c & \tilde{c} & p^2 \\ \bar{\psi}_\beta & \bar{\chi}_\beta & \tilde{\chi}^\beta & D^\beta \\ \psi_a^{-1/2} & \chi_a^{-1/2} & \tilde{\bar{\chi}}_a^{+1/2} & \bar{D}^{+1/2\,a} \\ \Lambda_{ab} & \eta_{ab} & \tilde{\eta}_{ab} & D^{ab} \\ \Lambda^{+-} & \eta & \tilde{\eta} & D^{-+} \\ \Lambda^{-a} & \eta^{-a} & \tilde{\eta}^{+a} & D^{+a} \end{bmatrix}.$$

It will be useful in the following to give the common name $\zeta$ to all these variables:

$$\zeta = \Big(\alpha, c, \tilde{c}; \bar{\psi}_\beta, \bar{\chi}_\beta, \tilde{\chi}^\beta; \psi_a^{-1/2}, \chi_a^{-1/2}, \tilde{\bar{\chi}}_a^{+1/2};$$
$$\Lambda_{ab}, \eta_{ab}, \tilde{\eta}_{ab}; \Lambda^{+-}, \eta, \tilde{\eta}; \Lambda^{-a}, \eta^{-a}, \tilde{\eta}^{+a}\Big). \tag{6.4}$$

Let us note that the part $Q_0$ of (6.2) *almost* coincides with the *naive* BRST charge of the system (2.14) which ignores the reducibility of the constraints $(\not{p}\bar{D})_\alpha$*:

$$Q_{\text{naive}} = \alpha^{-1}\left[cp^2 + i\left(c\frac{\partial}{\partial c} - \frac{1}{2}\right)\frac{\partial}{\partial \tilde{c}}\right]$$
$$+ i\frac{\partial}{\partial \alpha}\frac{\partial}{\partial \tilde{c}} + \bar{\chi}_\beta D^\beta + \chi^\beta(\not{p}\bar{D})_\beta$$
$$- \left(\chi^\beta \bar{\chi}_\beta\right)\alpha\frac{\partial}{\partial c} - \frac{\partial}{\partial \bar{\psi}_\beta}\frac{\partial}{\partial \tilde{\chi}^\beta} - \frac{\partial}{\partial \psi_\beta}\frac{\partial}{\partial \tilde{\bar{\chi}}^\beta}. \tag{6.5}$$

As one can see, comparing (6.5) with (6.1)–(6.3), the correct procedure produces two kind of effects:

(a) In place of the ghosts $\chi^\beta$, corresponding to the reducible set of constraints

$$(\not{p}\bar{D})_\beta,$$

one must use the ghosts $\chi_a^{-1/2}$ corresponding to the irreducible constraints

$$\bar{D}_a^{+1/2}.$$

* The analog *naive* result for the GS superstring can be found in ref. [26].

The two sets of ghosts are in fact related by the following harmonic projection:

$$\chi_a^{-1/2} = \frac{2}{p^+}\left(v^{+1/2}\sigma_a \not{p}\chi\right).$$

(b) A new part $Q_{\text{harmonic}}$ (6.3) arises in $Q_{\text{BRST}}$ (6.1) accounting for the purely harmonic constraints. Since the latter generate a Lie algebra (3.9), eq. (6.3) has the standard form of the BRST charge for a non-abelian gauge theory [4, 5].

Therefore, even though the functional dependence of the constraints $\not{p}\bar{d}$ (2.12) has a nontrivial effect on $Q_{\text{BRST}}$ (and requires a lot of technical ingenuity to treat) the end result turns out remarkably simple.

Starting with the $Q_{\text{BRST}}$ (6.1) we are now able to write down a covariant unconstrained superfield action for the linearized $D = 10$ IIB supergravity along the lines of the Neveu-West approach [17].

Choosing a BFV gauge function $\Psi = \partial/\partial c$, the first quantized BFV hamiltonian [4]

$$H_{\text{BFV}} = \left\{ Q_{\text{BRST}}, \frac{\partial}{\partial c} \right\} = \alpha^{-1}\left[ p^2 + i\frac{\partial}{\partial c}\frac{\partial}{\partial \tilde{c}} \right]$$

has the same form as the one of the ordinary bosonic particle.

Accordingly, we find the following second-quantized BRST action:

$$\begin{aligned} S_{\text{BRST}} = &\int \mathrm{d}\tau\,\mathrm{d}Z\,\mathrm{d}\zeta \left[\hat{K}(p^+)^2\Phi(\tau, Z, \zeta)\right] \\ &\times\left[ i\,\partial_\tau - \frac{1}{\alpha}\left( p^2 + i\frac{\partial}{\partial c}\frac{\partial}{\partial \tilde{c}} \right)\right]\left[(p^+)^2\Phi(\tau, Z, \zeta)\right], \\ Z \equiv &\,(z, u, v). \end{aligned} \tag{6.6}$$

The operator $\hat{K}$ acts on $\zeta$-coordinates (6.4) only by changing the signs of all the Lagrange multipliers and of the bosonic ghosts $\chi_a^{-1/2}$:

$$\begin{aligned} \hat{K}\Phi(\tau, Z, \zeta) = \Phi\big(&\tau, z; -\alpha, \ldots; -\bar{\psi}_\beta, \ldots; -\psi_a^{-1/2}, -\chi_a^{-1/2}, \ldots; \\ &-\Lambda_{ab}, \ldots; -\Lambda^{+-}, \ldots; -\Lambda^{+a}, \ldots\big). \end{aligned} \tag{6.7}$$

The two factors $(p^+)^2$ (recall that $p^+ \equiv v^{+1/2}/pv^{+1/2}$ is Lorentz invariant because the superscript + is internal) in (6.6) are needed to compensate for the SO(1, 1) charge $(-4)$ of the measure $\mathrm{d}\zeta$.

The action (6.6) is invariant under the (second-quantized) BRST-transformation:

$$\delta_{\text{BRST}}\Phi(\tau, Z, \zeta) = \Lambda Q_{\text{BRST}}\Phi(\tau, Z, \zeta) \tag{6.8}$$

due to the operator identity:

$$\hat{K}Q_{\text{BRST}} = -(Q_{\text{BRST}})^{\mathrm{T}}\hat{K}. \tag{6.9}$$

The superscript "T" means operator transposition and $\Lambda$ is a hermitian and anticommuting global parameter.

We impose the following covariant off-shell reality condition:

$$\hat{\Phi}^{*}(\tau, Z, \zeta) = \hat{F}\hat{K}\Phi(\tau, Z, \zeta). \tag{6.10}$$

$\hat{F}$ denotes the operator of grassmannian Fourier transform of exactly the same form as in eq. (5.27) (i.e. $\hat{F}$ acts only on the original harmonic superspace coordinates $(x^{\mu}, \theta_{\alpha}, \bar{\theta}_{\alpha}, u, v)$).

Let us note that by construction:

$$[Q_{\mathrm{BRST}}, \hat{F}] = 0. \tag{6.11}$$

This, and (6.9), insures the consistency of the reality condition (6.10) with the symmetries (6.8).

One can now further simplify our action by taking the Fourier transform with respect to those variables which change sign under the action of $\hat{K}$ (6.7):

$$\begin{aligned}
&\Phi\left(\tau, Z; \alpha,\ldots; \bar{\psi}_{\beta},\ldots \psi_{a}^{-1/2}, \chi_{a}^{-1/2},\ldots; \Lambda_{ab},\ldots; \Lambda^{+-},\ldots; \Lambda^{-a},\ldots\right)\\
&\quad = (p^{+})^{-8}\sqrt{\frac{\alpha}{2\pi}}\int \mathrm{d}y\,\mathrm{d}^{16}\phi^{\alpha}\,\mathrm{d}^{8}\bar{\phi}_{a}^{+1/2}\,\mathrm{d}^{8}\bar{\rho}_{a}^{-1/2}\,\mathrm{d}Y_{ab}\mathrm{d}Y^{-+}\,\mathrm{d}^{8}Y^{+a}\\
&\qquad \times\exp\left\{i\alpha y + \bar{\psi}_{\alpha}\phi^{\alpha} + \psi_{a}^{-1/2}\bar{\phi}^{+1/2\,a} + ip^{+}\chi_{a}^{-1/2}\bar{\rho}^{-1/2\,a}\right.\\
&\qquad\qquad \left. + i\Lambda^{ab}Y_{ab} + i\Lambda^{+-}Y^{-+} + i\Lambda^{-a}Y^{+a}\right\}\\
&\qquad \times\tilde{\Phi}\left(\tau, Z; y,\ldots,\phi^{\alpha},\ldots,\bar{\phi}_{a}^{+1/2}, \bar{\rho}_{a}^{-1/2},\ldots, Y_{ab},\ldots, Y^{-+},\ldots, Y^{+a},\ldots\right).
\end{aligned}$$

Then the action (6.6) acquires the form:

$$\begin{aligned}
S_{\mathrm{BRST}} = &\int \mathrm{d}\tau\,\mathrm{d}Z\,\mathrm{d}\tilde{\zeta}\left[(p^{+})^{-2}\tilde{\Phi}(\tau, Z, \tilde{\zeta})\right]\\
&\times\left[-\frac{\partial}{\partial y}\frac{\partial}{\partial \tau} - p^{2} - i\frac{\partial}{\partial c}\frac{\partial}{\partial \bar{c}}\right)\right]\left[(p^{+})^{-2}\tilde{\Phi}(\tau, Z, \tilde{\zeta})\right]
\end{aligned} \tag{6.12}$$

and the reality condition (6.10) becomes:

$$\tilde{\Phi}^{*}(\tau, Z, \tilde{\zeta}) = \hat{F}\tilde{\Phi}(\tau, Z, \tilde{\zeta}). \tag{6.13}$$

The construction of a covariant unconstrained superfield action for the linearized $D = 10$ IIB supergravity is an interesting result as it circumvents an existing no-go theorem [29]. The loophole which allows us to avoid this no go theorem can be traced to the fact that the "ghost-haunted" harmonic superfield $\Phi(\tau, Z, \zeta)$, while describing on shell a finite number of degrees of freedom, contains off shell an infinite number of gauge and auxiliary superfields. Analogous no-go theorems were circumvented in the past by the $D = 4$ harmonic superspace approach [20].

In eq. (6.12) we observe the existence of a few Parisi-Sourlas [27, 30] symmetries under OSp(1,1|2) rotations [17, 25, 31] in the subspaces parametrized respectively by:

$$
\begin{aligned}
&(\tau, y; c, \tilde{c}), \\
&(\bar{\chi}_\alpha, \tilde{\chi}^\alpha; \bar{\theta}_\alpha, \phi^\alpha), && \text{for each } a, \\
&(\bar{\rho}_a^{-1/2}, \tilde{\bar{\chi}}_a^{+1/2}; \theta_a^{-1/2} \bar{\phi}_a^{+1/2}), && \text{for each } a.
\end{aligned}
$$

($\theta_a^{-1/2}$ was defined in eq. (5.22).) Note that the SO(1,9) and SO(8) indices are internal with respect to the OSp(1,1|2) rotations. Thus, after Parisi-Sourlas dimensional reduction [27, 28, 30, 31] we get the reduced BRST action:

$$
\begin{aligned}
S_{\mathrm{BRST}}^{(\mathrm{red})} &= \int \mathrm{d}^{10}p\, \mathrm{d}^8\theta_a^{+1/2} \mathrm{d}u\, \mathrm{d}v \mathrm{d}Y_{ab} \mathrm{d}\eta_{ab} \mathrm{d}\tilde{\eta}_{ab} \mathrm{d}Y^{-+}\, \mathrm{d}\eta\, \mathrm{d}\tilde{\eta}\, \mathrm{d}Y_a^+\, \mathrm{d}\eta_a^-\, \mathrm{d}\tilde{\eta}_a^+ \\
&\quad \times \left[(p^+)^{-2} \tilde{\Phi}^{\mathrm{red}}\right] p^2 \left[(p^+)^{-2} \tilde{\Phi}^{\mathrm{red}}\right], \\
\tilde{\Phi}^{\mathrm{red}} &= \tilde{\Phi}^{\mathrm{red}}\left(p, \theta^{+1/2}, u, v, Y_{ab}, \dots, \tilde{\eta}_a^-\right),
\end{aligned}
\tag{6.14}
$$

which is invariant under the reduced (second quantized) BRST transformation:

$$
\begin{aligned}
&\delta_{\mathrm{BRST}} \tilde{\Phi}^{\mathrm{red}} = \Lambda Q_{\mathrm{BRST}}^{(\mathrm{red})} \tilde{\Phi}^{\mathrm{red}}, \\
&\tilde{Q}_{\mathrm{BRST}}^{(\mathrm{red})} = i\eta_{ab} \left( \tilde{D}^{ab} + \eta_d^a \frac{\partial}{\partial \eta_{bd}} - \eta_d^b \frac{\partial}{\partial \eta_{ad}} + \eta^{-a} \frac{\partial}{\partial \eta_b^-} - \eta^{-b} \frac{\partial}{\partial \eta_a^-} \right) \\
&\qquad + i\eta \left( \tilde{D}^{-+} - \eta^{-a} \frac{\partial}{\partial \eta^{-a}} \right) \\
&\qquad + i\eta_a^- D^{+a} - Y^{ab} \frac{\partial}{\partial \tilde{\eta}^{ab}} - Y^{-+} \frac{\partial}{\partial \tilde{\eta}} - Y^{+a} \frac{\partial}{\partial \tilde{\eta}^{+a}}.
\end{aligned}
\tag{6.15}
$$

Here $\tilde{D}^{ab}$, $\tilde{D}^{-+}$ are the same as in (5.23). We didn't find yet an explicit realization of the further Parisi-Sourlas OSp(1,1|2) symmetries acting on the subspaces spanned by the harmonics and ghosts, antighosts and (Fourier transformed) Lagrange multipliers associated with the harmonic constraints. The difficulty is with the constrained nature of the harmonics (3.4).

Nevertheless we can verify that the field equations of motion corresponding to (6.14):

$$
p^2 \tilde{\Phi}^{\mathrm{red}} = 0,
$$

together with the physical state conditions [32] ($G$ is the ghost-number operator):

$$\tilde{Q}^{(\mathrm{red})}_{\mathrm{BRST}}\tilde{\Phi}^{(\mathrm{red})}=0\,,$$

$$G\tilde{\Phi}^{(\mathrm{red})}=0\,,$$

$$\tilde{\Phi}^{(\mathrm{red})}\sim\tilde{\Phi}^{(\mathrm{red})}+\tilde{Q}^{(\mathrm{red})}_{\mathrm{BRST}}\tilde{\Phi}'\qquad\text{for any }\tilde{\Phi}',$$

yield the same Lorentz-covariant solution $\hat{\Phi}(z,u,v)$ (5.25), (5.26) as for the Dirac constrained equations (5.1)–(5.6). This can be easily shown by repeating the steps of ref. [17]. Here:

$$\hat{\Phi}(z,u,v)=\tilde{\Phi}^{(\mathrm{red})}_0\left(p,\theta^{+1/2},u,v,Y_{ab},Y^{-+},Y^+_a\right),$$

$$\frac{\partial}{\partial Y_{ab}}\tilde{\Phi}^{(\mathrm{red})}_0=\frac{\partial}{\partial Y^{-+}}\tilde{\Phi}^{(\mathrm{red})}_0=\frac{\partial}{\partial Y^+_a}\tilde{\Phi}^{(\mathrm{red})}_0=0$$

and $\tilde{\Phi}^{(\mathrm{red})}_0$ is the zeroth order term in the ghost-expansion of $\tilde{\Phi}^{(\mathrm{red})}$.

## 7. Conclusions

In the present paper we succeeded to reformulate the $D=10$, $N=2$ BS superparticle as a constrained system possessing Lorentz-covariant and functionally independent first-class constraints only.

The key ingredient of our formalism was the introduction of additional (pure gauge) bosonic degrees of freedom – Lorentz vector – and Lorentz spinor harmonics corresponding to the homogenous space $SO(1,9)/(SO(8)\times SO(1,1))$.

Unlike the previously proposed $D=10$ light-cone harmonic superspace [21], the present light-like vectors $u^{\pm}_{\mu}$ are constructed as bilinear composites of the Lorentz-spinor harmonics $v^{\pm 1/2}_{\alpha}$. Also the SO(8) indices of the space-like vectors $u^a_{\mu}$ are SO(8) ($s$)-spinorial rather than SO(8)-vector ones.

The last two properties of the above harmonics are crucial in two contexts:

(a) In the Lorentz-covariant separation of the set of functionally independent first class constraints.

(b) In proving the physical equivalence of the covariant N = 2 harmonic superparticle with the standard $N=2$ BS superparticle treated in the light-cone formalism.

We also succeeded to find a covariant unconstrained superfield action of the $D=10$ linearized IIB supergravity. The construction of the complete nonlinear action within the Neveu-West approach is, of course, a notorious cumbersome task. To perform it one would have to find simultaneously both the infinite number of higher nonlinear terms in the BRST action (6.6) as well as the infinite number of higher nonlinear terms in the BRST transformation law (6.8) in such a way that the complete action remains BRST-invariant.

On the other hand, we know that the non-covariant type II superstring field action contains no more than cubic interaction terms [33]. Therefore, the generalization of the present harmonic superspace formalism to the case of the GS superstring may be expected to provide the correct tractable framework for a complete derivation of the covariant interacting type II superstring field action.

The immediate task towards the extension of our results to the GS superstring is to identify the right combinations of the fermionic constraints (analog to (4.1)) which permit the reduction of the system to a first-class one by the elimination of the analog of the "gauge fixing condition" (4.4). This task is now under way.

It is amusing that the present work gives a happy illustration of the previously recognized fact [4] that the rank, level and class of a constrained system are not absolute properties, but depend on the choice of variables. The success of our treatment is due to the fact that using harmonic superspace variables we reduce the rank to one, the level to zero and the class to first which are the values for the usual simple gauge systems.

We are grateful for the warm hospitality and the stimulating atmosphere of the CERN Theory Division where this work was initiated. Two of us (E.N. and S.P.) are deeply indebted to E. Sokatchev and S. Kalitzin for numerous illuminating discussions and for teaching us the fundamentals of harmonic superspace. It is a pleasure for E.N. and S.P. to thank also the Weizmann Institute of Science, Rehovot, for most cordial hospitality. One of us (S.S) would like to thank J-W van Holten and Y. Eisenberg for very instructive discussions.

## Appendix A

### D = 10 AND D = 8 SPINOR CONVENTIONS

The $D=10$ $\gamma$-matrices and $D=10$ charge conjugation matrix are taken in the following representation:

$$\Gamma^{\mu}=\begin{pmatrix} 0 & (\sigma^{\mu})_{\alpha}^{\ \dot\beta} \\ (\tilde\sigma^{\mu})_{\dot\alpha}^{\ \beta} & 0 \end{pmatrix},$$

$$C_{10}=\begin{pmatrix} 0 & C^{\alpha\dot\beta} \\ (-C)^{\dot\alpha\beta} & 0 \end{pmatrix},$$

$$\Gamma^{11}\equiv\Gamma^{0}\Gamma^{1}\dots\Gamma^{9}=\begin{pmatrix} \delta_{\alpha}^{\beta} & 0 \\ 0 & -\delta_{\dot\alpha}^{\dot\beta} \end{pmatrix}.$$

Indices of $D=10$ left- (right-) handed MW spinors $\phi_\alpha$, $\psi_{\dot\alpha}$ are raised by means of $C_{10}$:

$$\phi^{\dot\alpha} = (-C)^{\dot\alpha\beta}\phi_\beta ,$$

$$\psi^{\alpha} = C^{\alpha\dot\beta}\psi_{\dot\beta} .$$

Throughout the paper we use $D=10$ $\sigma$-matrices with undotted indices only:

$$(\sigma^\mu)^{\alpha\beta} = C^{\alpha\dot\alpha}(\tilde\sigma^\mu)_{\dot\alpha}^{\ \beta} ,$$

$$(\sigma^\mu)_{\alpha\beta} = (-C)^{-1}_{\beta\dot\beta}(\sigma^\mu)_{\alpha}^{\ \dot\beta} ,$$

$$(\sigma^\mu)_{\alpha\gamma}(\sigma^\nu)^{\gamma\beta} + (\sigma^\nu)_{\alpha\gamma}(\sigma^\mu)^{\gamma\beta} = -2\delta_\alpha^\beta\eta^{\mu\nu} ,$$

$$\eta_{\mu\nu} = \mathrm{diag}(-,+,\dots,+) .$$

The standard basis in the space of the $\gamma$-matrices is

$$\Gamma^{\mu_1\dots\mu_n} \equiv \Gamma^{[\mu_1}\Gamma^{\mu_2}\dots\Gamma^{\mu_n]}, \qquad n=0,1,\dots,10,$$

where the square brackets denote antisymmetrization with respect to the enclosed indices. These matrices have the following properties:

$$\left(\Gamma^{\mu_1\dots\mu_{2r+1}}C_{10}^{-1}\right)^{\mathrm{T}} = (-1)^r\Gamma^{\mu_1\dots\mu_{2r+1}}C_{10}^{-1} ,$$

$$\left(C_{10}^{-1}\Gamma^{\mu_1\dots\mu_{2r}}\right)^{\mathrm{T}} = (-1)^{r+1}C_{10}\Gamma^{\mu_1\dots\mu_{2r}} ,$$

$$\Gamma^{\mu_1\dots\mu_n}\Gamma^{11} = (-1)^{n(n-1)/2}\frac{1}{(10-n)!}\varepsilon^{\mu_1\dots\mu_n\nu_1\dots\nu_{10-n}}\Gamma_{\nu_1\dots\nu_{10-n}}$$

$$\times\mathrm{tr}\left(\Gamma^{\mu_1\dots\mu_n}\Gamma^{\nu_1\dots\nu_m}\right) = -16\delta^{nm}(-1)^{1/2[n]}\det\begin{pmatrix} \eta^{\mu_1\nu_1} & \dots & \eta^{\mu_1\nu_n} \\ \dots & \dots & \dots \\ \eta^{\mu_n\nu_1} & \dots & \eta^{\mu_n\nu_n} \end{pmatrix} . \tag{A.1}$$

where $[n]=n$ for $n=$ even and $[n]=n+1$ for $n=$ odd.

Eqs. (A.1) imply for the $\sigma$ matrices $\sigma^{\mu_1\dots\mu_n} \equiv \sigma^{[\mu_1}\dots\sigma^{\mu_n]}$:

$$\left(\sigma_{\mu_1\dots\mu_{2r+1}}\right)^{\alpha\beta} = (-1)^r\left(\sigma_{\mu_1\dots\mu_{2r+1}}\right)^{\beta\alpha} ; \tag{A.2}$$

$$\left(\sigma^{\mu_1\dots\mu_{2r+1}}\right)_{\alpha\beta} = (-1)^{r+1}\frac{1}{(9-2r)!}\varepsilon^{\mu_1\dots\mu_{2r+1}\nu_1\dots\nu_{9-2r}}\left(\sigma_{\nu_1\dots\mu_{9-2r}}\right)_{\alpha\beta} ,$$

$$\left(\sigma^{\mu_1\dots\mu_{2r+1}}\right)^{\alpha\beta} = (-1)^r\frac{1}{(9-2r)!}\varepsilon^{\mu_1\dots\mu_{2r+1}\nu_1\dots\nu_{9-2r}}\left(\sigma_{\nu_1\dots\mu_{9-2r}}\right)^{\alpha\beta} . \tag{A.3}$$

Accounting for (A.2), (A.3) and (A.1) any (anti-)symmetric matrix $A^{\text{sym}}$ ($A^{\text{asym}}$) on the $D=10$ spinor space can be decomposed as follows:

$$A^{\text{sym}}_{\alpha\beta} = \sum_{[\mu]} A_{[\mu]}\sigma^{[\mu]}_{\alpha\beta}, \tag{A.4}$$

where $[\mu] = \mu, [\mu_1 \dots \mu_5]$,

$$A^{\text{asym}}_{\alpha\beta} = A_{[\mu_1\mu_2\mu_3]}\sigma^{\mu_1\mu_2\mu_3}_{\alpha\beta},$$

$$A_\mu = -\tfrac{1}{16} A^{\text{sym}}_{\alpha\beta}\sigma^{\alpha\beta}_\mu,$$

$$A_{[\mu_1\mu_2\mu_3]} = -\frac{1}{16(3!)} A^{\text{asym}}_{\alpha\beta}\sigma^{\alpha\beta}_{\mu_1\mu_2\mu_3},$$

$$A_{[\mu_1\dots\mu_5]} = -\frac{1}{32(5!)} A^{\text{sym}}_{\alpha\beta}\sigma^{\alpha\beta}_{\mu_1\dots\mu_5}.$$

Note that the coefficient $A_{[\mu_1\dots\mu_5]}$ is self-dual due to (A.3).

Let us also list the following useful properties of the $D=10$ $\sigma$-matrices:

$$(\sigma_\mu)^{\alpha\beta}(\sigma^\mu)^{\gamma\delta} + (\sigma_\mu)^{\beta\gamma}(\sigma^\mu)^{\alpha\delta} + (\sigma_\mu)^{\gamma\alpha}(\sigma^\mu)^{\beta\delta} = 0,$$

$$\sigma^\mu\sigma^{\nu_1\dots\nu_n} = \sigma^{\mu\nu_1\dots\nu_n} + \sum_{k=1}^{n}(-1)^k\eta^{\mu\nu_k}\sigma^{\nu_1\dots\hat{k}\dots\nu_n}, \tag{A.5}$$

where $\hat{k}$ means that the index $\nu_k$ is missing.

For the $D=8$ $\gamma$-matrices and $D=8$ charge conjugation matrix we use the following representation:

$$\Gamma^i_8 = \begin{pmatrix} 0 & (\gamma^i)_a^{\ \dot{b}} \\ (\tilde{\gamma}^i)_{\dot{a}}^{\ b} & 0 \end{pmatrix},$$

$$C_8 = \begin{pmatrix} C^{ab} & 0 \\ 0 & (-C)^{\dot{a}\dot{b}} \end{pmatrix},$$

$$C^{ab} = C^{ba}.$$

Indices of SO(8) $(s)$ and $(c)$ spinors $\phi_a$, $\psi_{\dot{a}}$ are raised as:

$$\phi^a = C^{ab}\phi_b, \qquad \psi^{\dot{a}} = (-C)^{\dot{a}\dot{b}}\psi_{\dot{b}}$$

## Appendix B

### ALGEBRAIC PROPERTIES OF HARMONIC EXPANSIONS

The harmonic expansion of a general field $\Phi$ (5.7) involves products of tensors:

$$u^{\pm}_{[\mu]} = v^{\pm 1/2}\sigma_{[\mu]}v^{\pm 1/2},$$
$$r_{[\mu]} = v^{+1/2}\sigma_{[\mu]}v^{-1/2},$$

for $[\mu] = \mu, [\mu_1\mu_2\mu_3], [\mu_1 \dots \mu_5]$. Note that

$$u^{\pm}_{[\mu_1\mu_2\mu_3]} \equiv 0$$

because of antisymmetry of $\sigma_{\mu_1\mu_2\mu_3}$ (A.2). Now, using the properties (A.5) of the $\sigma$-matrices one can easily verify the following identities for the traces in products of $u^{\pm}_{[\mu]}$ and/or $r_{[\mu]}$:

$$\begin{aligned} u^{\pm}_{\mu} u^{\pm\mu} &= 0\,, \\ u^{\pm}_{\mu} u^{\pm\mu\nu_1\dots\nu_4} &= 0\,, \\ r_{\mu} u^{\pm\mu\nu_1\dots\nu_4} &= -\tfrac{1}{2} u^{\pm}_{\mu} r^{\mu\nu_1\dots\nu_4} \mp \frac{4!}{2} u^{\pm[\nu_1} r^{\nu_2\nu_3\nu_4]}\,, \\ u^{\pm}_{\mu} r^{\mu\nu\lambda} &= \pm\left(r^{\nu}u^{\pm\lambda} - r^{\lambda}u^{\pm\nu}\right). \end{aligned} \tag{B.1}$$

Similarly, from (A.5) one easily gets that all further traces:

$$u^{\pm}_{\mu\nu_1\dots\nu_4} u^{\pm\mu\lambda_1\dots\lambda_4},$$
$$r_{\mu\nu_1\nu_2} u^{\pm\mu\lambda_1\dots\lambda_4},$$
$$r_{\mu\nu_1\dots\nu_4} u^{\pm\mu\lambda_1\dots\lambda_4},$$

are expressed by products of two $u^{\pm}_{[\mu]}$'s or of one $u^{\pm}_{[\mu]}$ and one $r_{[\mu]}$ with shorter index sets and without further traces. These identities lead to the restrictions (5.10) on the tensor coefficient fields in (5.7).

Finally, let us list the following important properties of the harmonic derivatives $D^{+a}$ (3.8) which are necessary for the proof of (5.11):

$$\begin{aligned} D^{+a}u^{+}_{[\mu]} &= 0\,, \\ D^{+a}u^{-}_{\mu} &= u^{a}_{\mu}\,, \\ D^{+a}u^{b}_{\mu} &= C^{ab}u^{+}_{\mu}\,, \\ D^{+a}u^{-}_{\mu_1\dots\mu_5} &= 5!\left(u^{a}_{\lambda}u^{+}_{[\mu_1}u^{-\lambda}_{\mu_2\dots\mu_5]} - u^{+}_{\lambda}u^{a}_{[\mu_1}u^{-\lambda}_{\mu_2\dots\mu_5]}\right), \\ D^{+a}r^{b} &= D^{+a}r^{b_1b_2b_3} = D^{+a}r^{b_1\dots b_5} = 0\,, \end{aligned} \tag{B.2}$$

where $r^{b_1\dots b_5} = u^{b_1}_{\mu_1}\dots u^{b_5}_{\mu_5}r^{\mu_1\dots\mu_5}$ and similarly for $r^{b}$, $r^{b_1b_2b_3}$.

## Appendix C

### EQUIVALENCE BETWEEN $2n$ REAL SECOND-CLASS AND $n$ HOLOMORPHIC FIRST-CLASS CONSTRAINTS

Let us consider a dynamical system which possesses a conjugated pair of fermionic* second class constraints $g_A$, $A = 1, 2$, satisfying:

$$\{g_A, g_B\}_{PB} = 2i\delta_{AB}. \tag{C.1}$$

By an appropriate canonical transformation one can always choose – at least locally – the phase-space coordinates ($\phi_1$, $\phi_2$, $X$, $p_{\phi_1}$, $p_{\phi_2}$, $P_X$), such that:

$$g_A = ip_{\phi_A} + \phi_A, \qquad A = 1, 2. \tag{C.2}$$

Then, introducing instead of the original real anticommuting coordinates and momenta, the holomorphic ones:

$$\begin{aligned}
\phi &= \sqrt{\tfrac{1}{2}}\,(\phi_1 + i\phi_2), \\
\bar{\phi} &= \sqrt{\tfrac{1}{2}}\,(\phi_1 - i\phi_2), \\
p_\phi &= \sqrt{\tfrac{1}{2}}\,(p_{\phi_1} + ip_{\phi_2}), \\
\bar{p}_\phi &= \sqrt{\tfrac{1}{2}}\,(p_{\phi_1} - ip_{\phi_2}), \\
\{\bar{p}_\phi, \phi\}_{\mathrm{PB}} &= \{p_\phi, \bar{\phi}\}_{\mathrm{PB}} = 1,
\end{aligned} \tag{C.3}$$

the relations (C.1) are written as:

$$\begin{aligned}
&\{g, g\}_{\mathrm{PB}} = \{\bar{g}, \bar{g}\}_{\mathrm{PB}} = 0, \\
&\{g, \bar{g}\}_{\mathrm{PB}} = 2i
\end{aligned} \tag{C.4}$$

where

$$\begin{aligned}
g &= \sqrt{\tfrac{1}{2}}\,(g_1 + ig_2) = ip_\phi + \phi, \\
\bar{g} &= \sqrt{\tfrac{1}{2}}\,(g_1 - ig_2) = i\bar{p}_\phi + \bar{\phi}.
\end{aligned}$$

Due to the second class constraints (C.2) we have to use in place of the Poisson

* The choice of the constraints to be fermionic is not essential. It is made here only to fit the application to the present context.

brackets, *Dirac brackets* (DB):

$$\{\phi_A, \phi_B\}_{\mathrm{DB}} = \tfrac{1}{2} i \delta_{AB},$$

which equivalently can be rewritten in the holomorphic coordinates (C.3) as:

$$\{\phi, \phi\}_{\mathrm{DB}} = \{\bar{\phi}, \bar{\phi}\}_{\mathrm{DB}} = 0,$$
$$\{\phi, \bar{\phi}\}_{\mathrm{DB}} = -\tfrac{1}{2} i. \tag{C.5}$$

Let us now demonstrate that the standard canonical quantization of the system in terms of the *Dirac brackets* (C.5) is equivalent to the canonically quantized system with *one holomorphic first-class* constraint:

$$g = 0, \tag{C.6}$$

whereby $\bar{g} = 0$ is discarded as an (antiholomorphic) gauge fixing condition, which, according to (C.4), fixes the gauge invariance associated with (C.6)*.

We start by treating the second-class system, then, we will treat the first-class one. In the end, we will compare the results and recognize their identity.

For the second-class system we write the quantized Dirac brackets (C.5):

$$\{\phi, \phi\} = \{\bar{\phi}, \bar{\phi}\} = 0,$$
$$\{\phi, \bar{\phi}\} = \tfrac{1}{2}.$$

They coincide (up to a factor of $\frac{1}{2}$) with the algebra of a pair of fermion creation and annihilation operators.

Then, the wave functions in the holomorphic representation [34] are of the form:

$$F(\phi, X) = F_0(X) + \phi F_1(X), \tag{C.7}$$

where $\phi$ is a complex anticommuting coordinate (such functions are called holomorphic).

Let us now quantize the system with the holomorphic first class constraint (C.6) *à la* Dirac:

$$\hat{g} F(\phi, \bar{\phi}, X) = \left( -\frac{\partial}{\partial \bar{\phi}} + \phi \right) F(\phi, \bar{\phi}, X) = 0.$$

One gets the general solution (recall that $\phi$ and $\bar{\phi}$ are a pair of complex conjugated anticommuting coordinates (C.3)):

$$F(\phi, \bar{\phi}, X) = \mathrm{e}^{\bar{\phi}\phi} F(\phi, X) = \mathrm{e}^{\bar{\phi}\phi} [F_0(X) + \phi F_1(X)]. \tag{C.8}$$

* This procedure can be straightforwardly generalized to the case when $g_A$ carry indices: $\{g_A^a, g_B^b\} = 2i\delta_{AB} C^{ab}$, $C^{ab} = C^{ba}$, $a, b = 1, \ldots, n$.

Thus, both (C.7) and (C.8) describe the same physical degrees of freedom by the two-component wave function $(F_0(X), F_1(X))$. The equivalence is proven.

We prefer in the present paper the method which uses first class constraints, because it allows us to take advantage of the powerful machinery of BFV-BRST which was tailored for such constraints.

Let us now show how the general discussion above applies to the *example of the* $N=2$ *superparticle*.

The following decomposition of the original MW constraints $d^{\alpha}_{A}$ (2.3) with the help of the harmonics (3.4):

$$d^{\alpha}_{A} = \sqrt{\frac{2}{p^{+}}}\,(\sigma^{a}v^{+1/2})^{\alpha} d_{Aa} + \frac{1}{\sqrt{2p^{+}}}\left(\not p\sigma^{+}\sigma^{a}v^{-1/2}\right)^{\alpha} g_{Aa} \tag{C.9}$$

covariantly separates the 16 *real* second-class constraints:

$$g^{a}_{A} = \frac{1}{\sqrt{2p^{+}}}\left(v^{-1/2}\sigma^{a}\sigma^{+}d_{A}\right), \tag{C.10}$$

from the 16 *real* first class constraints $d^{a}_{A}$:

$$d^{a}_{A} = \frac{1}{\sqrt{2p^{+}}}\left(v^{+1/2}\sigma^{a}\not p d_{A}\right). \tag{C.11}$$

(recall that $p^{+} \equiv v^{+1/2}\not p v^{+1/2}$ is Lorentz-invariant in spite of the misleading notation. This is because the superscript + is internal). The class of the constraints is verified by inspecting the Poisson rackets:

$$\left\{g^{a}_{A}, g^{b}_{B}\right\}_{\mathrm{PB}} = 2iC^{ab}\delta_{AB}, \tag{C.12}$$

$$\left\{d^{a}_{A}, d^{b}_{B}\right\}_{\mathrm{PB}} = -iC^{ab}\delta_{AB}p^{2},$$

$$\left\{d^{a}_{A}, g^{b}_{B}\right\}_{\mathrm{PB}} = 0. \tag{C.13}$$

The holomorphic constraints $g^{+1/2\,a}$, $\bar{g}^{-1/2\,a}$ and $d^{-1/2\,a}$, $\bar{d}^{-1/2\,a}$ (4.1)–(4.4), used in the main text, are simply expressed in terms of the real constraints (C.10)–(C.11):

$$g^{+1/2\,a} = \sqrt{\frac{p^{+}}{2}}\left[\frac{g^{a}_{1} + ig^{a}_{2}}{\sqrt{2}}\right] \equiv \sqrt{\frac{p^{+}}{2}}\,g^{a},$$

$$\bar{g}^{-1/2\,a} = \frac{1}{\sqrt{2p^{+}}}\left[\frac{g^{a}_{1} - ig^{a}_{2}}{\sqrt{2}}\right] \equiv \frac{1}{\sqrt{2p^{+}}}\bar{g}^{a},$$

$$d^{-1/2\,a} = \frac{1}{\sqrt{2p^{+}}}\left[\frac{d^{a}_{1} + id^{a}_{2}}{\sqrt{2}}\right] \equiv \frac{1}{\sqrt{2p^{+}}}d^{a},$$

$$\bar{d}^{+1/2\,a} = \sqrt{2p^{+}}\left[\frac{d^{a}_{1} - id^{a}_{2}}{\sqrt{2}}\right] \equiv \sqrt{2p^{+}}\,\bar{d}^{a}. \tag{C.14}$$

In order to rewrite (C.10) in the canonical form (C.2) we make the following change of variables $\theta_\alpha^A \to \phi_A^a, \psi_A^a$:

$$\theta_\alpha^A = \frac{1}{\sqrt{2p^+}}\left(\sigma^+\sigma^b v^{-1/2}\right)_\alpha \phi_{Ab} + \frac{1}{\sqrt{2p^+}}\left(\not{p}\sigma^b v^{+1/2}\right)_\alpha \psi_{Ab},$$

or, inversely:

$$\phi_A^a = \frac{1}{\sqrt{2p^+}}\left(v^{-1/2}\sigma^a\sigma^+\not{p}\theta^A\right),$$

$$\psi_A^a = \sqrt{\frac{2}{p^+}}\left(v^{+1/2}\sigma^a\theta^A\right). \tag{C.15}$$

Now we can use a new set of canonical coordinates:

$$\left(p_\mu, \phi_A^a, \psi_A^a, u, v\right) \tag{C.16}$$

instead of the old one $(x^\mu, \theta_\alpha^A, u, v)$*. In particular, the (anti-) holomorphic anti-commuting coordinates $\theta^{\pm 1/2\, a}$ introduced in (5.22) are expressed in terms of the real $\phi_A^a, \psi_A^a$ as:

$$\theta^{+1/2\, a} = \sqrt{\frac{p^+}{2}}\left[\frac{\phi_1^a + i\phi_2^a}{\sqrt{2}}\right] \equiv \sqrt{\frac{p^+}{2}}\,\phi^a,$$

$$\theta^{-1/2\, a} = \frac{1}{\sqrt{2p^+}}\left[\frac{\psi_1^a + i\psi_2^a}{\sqrt{2}}\right] \equiv \frac{1}{\sqrt{2p^+}}\psi^a. \tag{C.17}$$

The real constraints (C.10) and (C.11) and their (anti-) holomorphic linear combinations (C.14) take now the simple form:

$$g_A^a = ip_{\phi_A}^a + \phi_A^a,$$

$$d_A^a = ip_{\psi_A}^a - \tfrac{1}{2}p^2\psi_A^a,$$

$$g^a = ip_\phi^a + \phi^a,$$

$$\bar{g}^a = i\bar{p}_\phi^a + \bar{\phi}^a,$$

$$d^a = ip_\psi^a - \tfrac{1}{2}p^2\psi^a,$$

$$\bar{d}^a = i\bar{p}_\psi^a - \tfrac{1}{2}p^2\bar{\psi}^a, \tag{C.18}$$

* We shall not need the complicated explicit formulas expressing the new canonical momenta conjugated to (C.16) as functions of the old coordinates and momenta.

in terms of the holomorphic coordinates:

$$\phi^a = \sqrt{\tfrac{1}{2}}\left(\phi_1^a + i\phi_2^a\right),$$

$$\bar{\phi}^a = \sqrt{\tfrac{1}{2}}\left(\phi_1^a - i\phi_2^a\right),$$

$$\psi^a = \sqrt{\tfrac{1}{2}}\left(\psi_1^a + i\psi_2^a\right),$$

$$\bar{\psi}^a = \sqrt{\tfrac{1}{2}}\left(\psi_1^a - i\psi_2^a\right),$$

where $p_{\phi A}^a$, $\bar{p}_{\phi}^a$, $p_{\phi}^a$ are the canonical momenta conjugated to $\phi_A^a$, $\phi^a$, $\bar{\phi}^a$, respectively.

The Poisson brackets (C.12) become:

$$\{g^a, g^b\}_{\mathrm{PB}} = \{\bar{g}^a, \bar{g}^b\}_{\mathrm{PB}} = 0,$$

$$\{g^a, \bar{g}^b\}_{\mathrm{PB}} = 2iC^{ab}.$$

Correspondingly, we get the following Dirac brackets in the holomorphic representation:

$$\{\phi^a, \phi^b\}_{\mathrm{DB}} = \{\bar{\phi}^a, \bar{\phi}^b\}_{\mathrm{DB}} = 0,$$

$$\{\phi^a, \bar{\phi}^b\}_{\mathrm{DB}} = -\tfrac{1}{2}iC^{ab}. \tag{C.19}$$

Now, canonically quantizing the $N=2$ harmonic superparticle in terms of the Dirac brackets (C.19) and accounting for the additional first class constraints $d_A^a$ (C.11), $p^2$ and the harmonic ones $D^{ab}, D^{-+}, D^{+a}$, the solution for the wave function reads in the holomorphic representation (cf. eq. (C.7)):

$$F = F(p, \phi, u, v) = \sum_k \frac{1}{k!}\phi^{a_1}\dots\phi^{a_k}F_{a_1\dots a_k}(p, u, v),$$

$$p^2 F_{a_1\dots a_k}(p, u, v) = 0,$$

$$D^{+a}F_{a_1\dots a_k}(p, u, v) = 0. \tag{C.20}$$

Recalling (C.17), we conclude that the quantum solution (C.20) of the system with the second class constraints $g_A^a$ (C.10) is identical to the quantum solution (5.25), (5.26) of the system with holomorphic first-class constraints $g^{+1/2\,a}$ (4.3), (C.14).

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